Ray Optics 22

Microscope

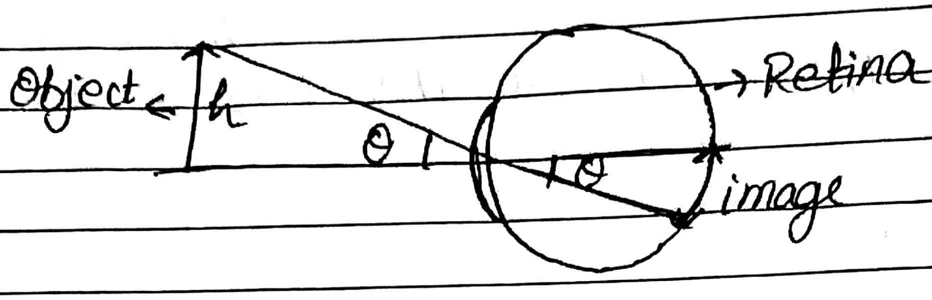
Purpose of microscope is to magnify a small object & form it's large image.

With naked eye:

To see a small object Clearly, we bring it closer to our eye. However, there is a minimum distance for clear vision of an object. This distance is called Least Distance of distinct vision (D).
For an average human, $D = 25cm$.

So with naked eye, a small object appears largest when it is at 25cm (D) from eye.

Visual Angle:



The size of object sensed by us is related to size of the image formed on the retina

The size of image on retion $\propto$ Angle subtended by Object at the eye
⇓
Visual Angle

Microscope increases this visual Angle.

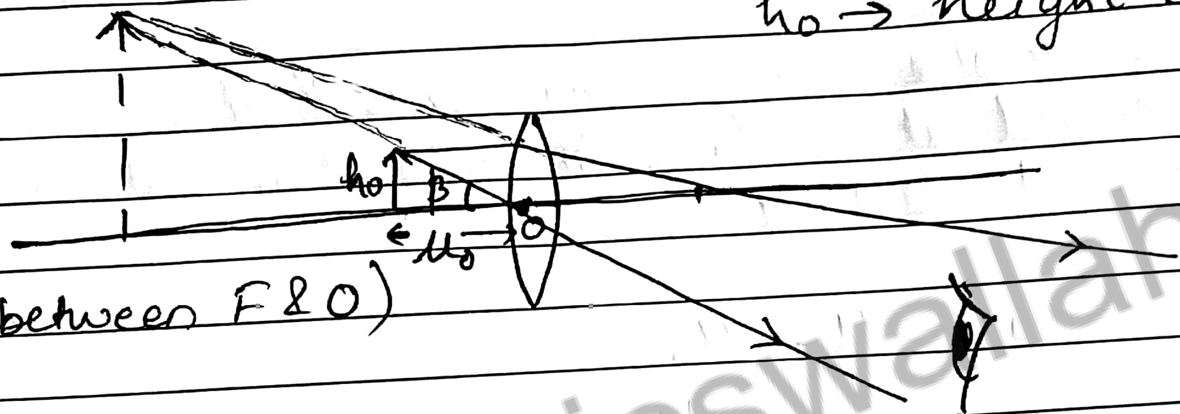
For a Microscope, Magnification (M)

$$M = \frac{\text{Visual angle formed by final image}}{\text{Visual angle formed by object kept at D}}$$

$$M = \frac{\beta}{\alpha}$$

Simple Microscope: → (A convex lens)

h_o → height of object



(Object between F & O)

β → visual angle formed by final image

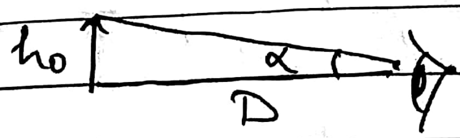
$$\tan\beta = \frac{h_o}{u_o}$$

∵ β is small (as h_o is small)

$\tan\beta \approx \beta$

$$\beta \approx \frac{h_o}{u_o}$$

α → visual angle formed by object kept at D



$$\tan\alpha = \frac{h_o}{D}$$ ($\tan\alpha \approx \alpha$)

$$\alpha = \frac{h_o}{D}$$

$$M = \frac{\beta}{\alpha} = \frac{h_o / u_o}{h_o / D}$$

$$\boxed{M = \frac{D}{u_o}}$$ → Simple Microscope

Now, to get maximum Magnification, β should be max.

So we make such arrangements that final image is formed at D. (Max magnification that can be viewed distinctly)

so $u = -u_o$
$v = -D$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{-D} - \frac{1}{-u_o}$$

$$\frac{1}{u_o} = \frac{1}{f} + \frac{1}{D}$$

$$M = \frac{D}{u_o} = D\left(\frac{1}{f} + \frac{1}{D}\right) = 1 + \frac{D}{f}$$

$$\boxed{M_D = 1 + \frac{D}{f}}$$ → Max Magnification.

But under Max magnification, the eyes are strained.

for relaxed eye, final image should be near ∞

$$u = -u_o$$
$$v = \infty$$
$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{\infty} - \frac{1}{-u_o}$$

$$\frac{1}{u_o} = \frac{1}{f}$$

$$M = \frac{D}{u_o} = \frac{D}{f}$$

$$\boxed{M_\infty = \frac{D}{f}}$$ → Magnification for relaxed eye

So summary for simple Microscope

$$\boxed{M = \frac{D}{u_o}}$$

- Max Magnification → $M_D = 1 + \frac{D}{f}$ strained eye
- Min Magnification → $M_\infty = \frac{D}{f}$ relaxed eye

$$\frac{D}{f} \leq M \leq 1 + \frac{D}{f}$$

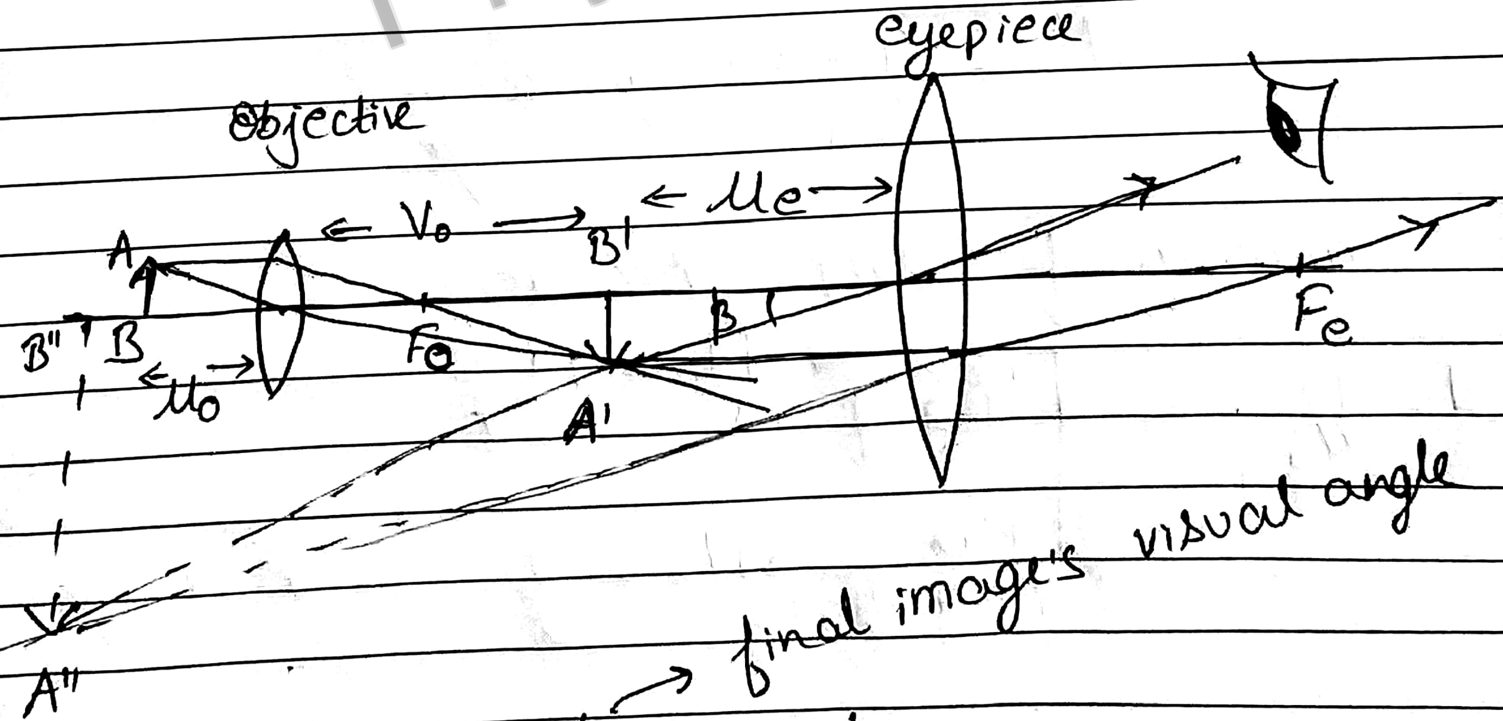
Note: M can be increased by decreasing f, but practically Simple Microscope can give $M = 9X$ or so, for further magnification we use a compound microscope

Compound Microscope: → Two Convex lens

i) Objective → Close to object → small focal length
ii) eyepiece → Close to eye → large focal length

Object is placed beyond first focus of objective which forms inverted image (magnified). This acts as virtual object for eyepiece. This virtual object lies between focus & optical centre of eyepiece & eyepiece acts like a simple microscope & forms a virtual erect & Magnified final image

$A \uparrow B$ h_o → object



→ final image's visual angle

$$\tan\beta \approx \beta \approx \frac{A'B'}{u_e}$$

for objective

$$m_o = \frac{h_i}{h_o} = \frac{-A'B'}{AB}$$

also $$m_o = \frac{v}{u} = \frac{V_o}{-u_o}$$

$$\Rightarrow \frac{+A'B'}{AB} = \frac{V_o}{+u_o}$$

$$A'B' = \frac{V_o}{u_o} AB$$ Let $AB = h_o$

$$A'B' = \frac{V_o}{u_o} AB$$

$$\beta = \frac{A'B'}{u_e} = \frac{V_o}{u_o}\frac{AB}{u_e} = \frac{V_o}{u_o}\frac{h_o}{u_e}$$

$\alpha \rightarrow$ Visual angle formed by object kept at D

$$\tan\alpha \simeq \alpha = \frac{h_o}{D}$$

$$M = \frac{\beta}{\alpha} = \frac{\frac{V_o}{u_o}\frac{h_o}{u_e}}{\frac{h_o}{D}} = \frac{V_o}{u_o}\frac{D}{u_e}$$

$$M = \left(\frac{v_o}{u_o}\right)\left(\frac{D}{u_e}\right)$$

if you look closely $\left(\frac{v_o}{u_o}\right) = m_o$ $\left(\frac{D}{u_e}\right) = m_e$

eyepiece acts like a simple Microscope

$\boxed{M = m_o \times m_e}$ → remember → (simple microscope m_e)

length of Compound Microscope

$\boxed{l = |v_o| + |u_e|}$

Now, for

i) Maximum magnification: (strained eye) $m_e = 1 + \frac{D}{f_e}$

final image is formed at D by eyepie.

we get

$$\frac{1}{u_e} = \frac{1}{D} + \frac{1}{f_e} \Rightarrow \frac{D}{u_e} = 1 + \frac{D}{f_e}$$

$$\boxed{M_D = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)}$$

ii) Relaxed eye: Minimum Magnification $m_e = \frac{D}{f_e}$

final image is formed at ∞ by eyepiece

we get $\frac{1}{u_e} = \frac{1}{f_e} \Rightarrow \frac{D}{u_e} = \frac{D}{f_e}$

$$\boxed{M_\infty = \frac{v_o}{u_o}\left(\frac{D}{f_e}\right)}$$ $$\boxed{l = |v_o| + |f_e|}$$

Also remember M is -ve as final image is inverted

Q1) In a Compound microscope, focal length of Objective lens is 1.2cm & focal length of eye piece is 3.0cm. When object is kept at 1.25cm infront of objective, final image is formed at ∞. Magnifying power is :

a) 200 b) 100 c) 400 d) 150

Solution :

$$M = m_o \times m_e$$

$$M = \left(\frac{V_o}{u_o}\right)\left(\frac{D}{u_e}\right)$$

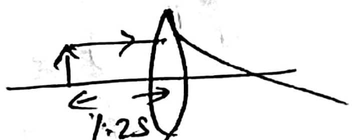
for final image at ∞ $m_e = \frac{D}{f_e}$

$$M = \left(\frac{V_o}{u_o}\right)\left(\frac{D}{f_e}\right)$$

$u_o = 1.25cm$
$D = 25cm$
$f_e = 3.0 cm$
$V_o = ?$
$f_o = 1.2cm$

for objective

$$\frac{1}{f_o} = \frac{1}{V} - \frac{1}{u}$$



Objective

$u = -1.25cm$

$$\frac{1}{1.2} = \frac{1}{V} - \frac{1}{(-1.25)}$$

$$\frac{1}{V} = \frac{1}{1.2} - \frac{1}{1.25}$$

$$\frac{1}{V} = \frac{10}{12} - \frac{100}{125}$$

$$\frac{1}{V} = \frac{5}{6} - \frac{4}{5}$$

$$\frac{1}{V} = \frac{25 - 24}{30} = \frac{1}{30}$$

$V = 30 \Rightarrow V_o = 30$

$$M = \frac{V_o}{u_o}\left(\frac{D}{f_e}\right)$$

$$= \frac{30}{1.25} \times \frac{25}{3}$$

$$= 200$$

Q2) A compound microscope consists of an objective lens of focal length 2.0cm & an eye-piece of focal length 6.25cm. The length of microscope is 15cm. How far from the objective should an object be placed in order to obtain final image at least distance of distinct vision. Also calculate Magnifying Power.

<u>Solution</u>

$f_o = 2.0cm$

$f_e = 6.25cm$

$l = 15cm \Rightarrow |v_o| + |u_e| = 15cm$

final image is at 25cm from eyepiece

for eyepiece

$v = -25cm$

$f = 6.25cm$

$u = ?$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{6.25} = \frac{1}{-25} - \frac{1}{u}$$

$$-\frac{1}{u} = \frac{1}{6.25} + \frac{1}{25}$$

$$-\frac{1}{u} = \frac{100}{625} + \frac{1}{25}$$

$$-\frac{1}{u} = \frac{4}{25} + \frac{1}{25}$$

$$-\frac{1}{u} = \frac{5}{25}$$

$u = -5$

$u = -u_e$

$\Rightarrow u_e = 5cm$

$|V_o| + |u_e| = 15$

$V_o = 15 - 5 = 10\,cm$

for objective

$v = 10\,cm$

$f = 2.0\,cm$

$u = ?$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{2} = \frac{1}{10} - \frac{1}{u}$$

$$\frac{1}{u} = \frac{1}{10} - \frac{1}{2} = \frac{2-10}{20}$$

$$\frac{1}{u} = \frac{-8}{20}$$

$$u = \frac{-5}{2} = -2.5\,cm$$

$u = -u_o = -2.5\,cm \qquad u_o = 2.5\,cm$

$M = m_o \times m_e$

$= \left(\frac{V_o}{u_o}\right)\left(\frac{D}{u_e}\right)$ for final image at D $m_e = 1 + \frac{D}{f_e}$

$= \left(\frac{V_o}{u_o}\right)\left(1 + \frac{D}{f_e}\right)$

$= \left(\frac{10}{2.5}\right)\left(1 + \frac{25}{6.25}\right) = \left(\frac{100}{25}\right)\left(1 + \frac{2500}{625}\right) = 20$